Chapter 15: Inferences, Explanations and Uncertainty

15.1 Opening Vignette: Konica Automates a Help Desk with Case-based Reasoning

The Problem

- **Konica Business Machines wanted to fully automate its help desk for**
  - **Internal Support**
  - **External Support**

The Solution

- Most Promising Approach
  - Case-based Reasoning
- Software Artistry's Expert Advisor
  - Could Run Multiple Problem Resolution Modes
    - Decision Trees
    - Adaptive Learning
    - Tech Search
    - More
  - Used 'standard cases initially
  - Later used real cases to boost accuracy
  - Includes digital photos

Expert Advisor

- Used by Internal Tech Support (6750 people)
- Used by Customers

- Tech Support handles unusual cases
- Early stage testing: 65% hit rate
- Adaptive learning being added

15.2 Reasoning in Artificial Intelligence

- **Once knowledge is acquired, it must be stored and processed (reasoned with)**
- **Need a computer program to access knowledge for making inferences**
- **This program is an algorithm that controls a reasoning process**
- **Inference engine or control program**
- **Rule interpreter (in rule-based systems)**
- **The inference engine directs the search through the knowledge base**

How People Reason and Solve Problems

Sources of Power

- Formal methods (logical deduction)
- Heuristic reasoning (IF-THEN rules)
- Focus--common sense related toward more or less specific goals
- Divide and conquer
- Parallelism

- **Representation**
- **Analogy**
- **Synergy**
- **Serendipity (Luck)**

(Lenat [1982])

Sources of power translated to specific reasoning or inference methods (Table 15.1)

Table 15.1 Reasoning Methods

Method	Description
Deductive reasoning	**Move from a general principle to a specific inference. General principle is composed of two or more premises.**
Inductive reasoning	**Move from some established facts to draw general conclusions.**
Analogical reasoning	**Derive answer to a question by known analogy. It is a verbalization of internalized learning process (Tuthill [1990] and Owen [1990]). Use of similar, past experiences.**
Formal reasoning	**Syntactic manipulation of data structure to deduce new facts, following prescribed rules of inferences (e.g., predicate calculus).**
Procedural (numeric) reasoning	**Use of mathematical models or simulation (e.g., model-based reasoning, qualitative reasoning and temporal reasoning--the ability to reason about the time relationships between events).**
Metalevel reasoning	**Knowledge about what you know (e.g., about the importance and relevance of certain facts and/or rules).**

Reasoning with Logic

- **Modus Ponens**
  - If A, then B
  - [A AND (A $\rightarrow$ B)] $\rightarrow$ B
  - A and (A $\rightarrow$ B) are propositions in a knowledge base

- **Modus Tollens**: when B is known to be false

- **Resolution**: combines substitution, modus ponens and other logical syllogisms

15.3 Inferencing with Rules: Forward and Backward Chaining

- Firing a rule: When all of the rule's hypotheses (the “if parts”) are satisfied
- Can check every rule in the knowledge base in a forward or backward direction
- Continues until no more rules can fire, or until a goal is achieved

Forward and Backward Chaining

– **Chaining: Linking a set of pertinent rules**

– **Search process: directed by a rule interpreter approach:**

  - **Forward chaining. If the premise clauses match the situation, then the process attempts to assert the conclusion**

  - **Backward chaining. If the current goal is to determine the correct conclusion, then the process attempts to determine whether the premise clauses (facts) match the situation**

Backward Chaining

- **Goal-driven - Start from a potential conclusion (hypothesis), then seek evidence that supports (or contradicts) it**

- **Often involves formulating and testing intermediate hypotheses (or subhypotheses)**

Forward Chaining

- **Data-driven - Start from available information as it becomes available, then try to draw conclusions**
- **What to Use?**
  - **If all facts available up front (as in auditing) - forward chaining**
  - **Diagnostic problems - backward chaining**

AIS In Focus 15.1: The Functions of the Inference Engine

1. Fire the rules

2. Present the user with questions

3. Add the answer to the ES "blackboard" (assertion base)

4. Infer a new fact from a rule

5. Add the inference fact to the blackboard

6. Match the blackboard to the rules

7. If there are any matches, fire rules

8. If there are two further matches, check to see if goal is reached

9. Fire the lowest numbered unfired rule

The program works through the knowledge base until it can post a fact (or a partial fact if certainty factors are being used) to the blackboard.

Once a fact has been posted, the system goes back to the knowledge base to infer more facts. This continues until the present goal is achieved or until all rules have been fired.

15.4 The Inference Tree

(Goal Tree or Logical Tree)

- **Schematic view of the inference process**
- **Similar to a decision tree (Figure 15.2)**
- **Inferencing: tree traversal**

- **Advantage: Guide for the *Why* and *How* Explanations**

15.5 Inferencing with Frames

- Much more complicated than reasoning with rules
- Slot provides for expectation-driven processing
- Empty slots can be filled with data that confirm expectations
- Look for confirmation of expectations
- Often involves filling in slot values

- Can Use Rules in Frames
- Hierarchical Reasoning

15.6 Model-based Reasoning

- Based on knowledge of structure and behavior of the devices the system is designed to understand
- Especially useful in diagnosing difficult equipment problems
- Can overcome some of the difficulties of rule-based ES (AIS in Action 15.2)
- Systems include a (deep-knowledge) model of the device to be diagnosed that is then used to identify the cause(s) of the equipment's failure
- Reasons from "first principles" (Common Sense)
- Often combined with other representation and inferencing methods.

AIS In Action 15.2: Model-Based ES Helps the Environment

- **Westinghouse Savannah River Company**
- **Project to develop a representation schema for engineering and common-sense knowledge about environmental and biological impacts of a nuclear weapons processing facility operations**
- **Learn by doing approach**
- **Cyc: a set of general knowledge about the world**
- **New concepts are added to the qualitative model**
- **Dynamic events can be described**
- **Relevant knowledge portions may be shared or reused**
- **Model-based ES can overcome some difficulties of rule-based ES**

- Model-based ES tend to be "transportable”
- Simulates the structure and function of the machinery being diagnosed
- Models can be either mathematical or component
- Necessary condition is the creation of a complete and accurate model of the system under study
- Especially useful in real-time systems

15.7 Case-based Reasoning (CBR)

- Adapt solutions used to solve old problems for new problems
- Variation - Rule-induction method (Chap. 13)
- But, CBR
  - Finds cases that solved problems similar to the current one, and
  - Adapts the previous solution or solutions to fit the current problem, while considering any difference between the two situations

Finding Relevant Cases Involves

- Characterizing the input problem, by assigning appropriate features to it
- Retrieving the cases with those features
- Picking the case(s) that best match the input best

- Extremely effective in complex cases
- Justification - Human thinking does not use logic (or reasoning from first principle)
- Process the right information retrieved at the right time

- Central problem - Identification of pertinent information whenever needed - Use Scripts

What is a Case?

- <u>Case</u> - Defines a problem in natural language descriptions and answers to questions, and associates with each situation a proper business action
- <u>Scripts</u> - Describe a well-known sequence of events
  - Often “reasoning is applying scripts”
  - More Scripts, Less (Real) Thinking
  - Can be constructed from historical cases
  - Case-based reasoning is the essence of how people reason from experience
  - CBR - a more psychologically plausible expert reasoning model than a rule-based model (Table 15.2)
- Advantages of CBR (Table 15.3)

TABLE 15.2 Comparison of Case-based and Rule-based Reasoning

Criterion	Rule-based Reasoning	Case-based Reasoning
Knowledge unit	Rule	Case
Granularity	Fine	Coarse
Knowledge acquisition units	Rules, hierarchies	Cases, hierarchies
Explanation mechanism	Backtrack of rule firings	Precedent cases
Characteristic output	Answer, plus confidence measure	Answer, plus precedent cases
Knowledge transfer across problems	High, if backtracking Low, if deterministic	Low
Speed as a function of knowledge base size	Exponential, if backtracking Linear, if deterministic	Logarithmic, if index tree balanced

Table 15.2 (continued)

Criterion	Rule-based Reasoning	Case-based Reasoning
Domain requirements	Domain vocabulary Good set of inference rules Either few rules or rules apply sequentially Domain mostly obeys rules	Domain vocabulary Database of example cases Stability--a modified good solution is probably still good Many exceptions to rules
Advantages	Flexible use of knowledge Potentially optimal answers	Rapid response Rapid knowledge acquisition Explanation by examples
Disadvantages	Computationally expensive Long development time Black-box answers	Suboptimal solutions Redundant knowledge base

Source: Courtesy of Marc Goodman, Cognitive Systems, Inc. Based on: M. Goodman, "PRISM: A Case-Based Telex Classifier," in A. Rappaport and R. Smith (eds.), *Innovative Applications of Artificial Intelligence*. Vol. 11, Cambridge, MA: MIT Press, 1990.

TABLE 15.3 Advantages of Case-based Reasoning

- **Knowledge acquisition is improved: easier to build, simpler to maintain, less expensive to develop and support**
- **System development time is faster**
- **Existing data and knowledge are leveraged**
- **Complete formalized domain knowledge (as is required with rules) is not required**
- **Experts feel better discussing concrete cases (not general rules)**
- **Explanation becomes easier. Rather than showing many rules, a logical sequence can be shown**
- **Acquisition of new cases is easy (can be automated; for an example of knowledge acquisition of cases, see diPiazza and Helsabeck [1990])**
- **Learning can occur from both successes and failures**

Case-based Reasoning Process (Figure 15.3)

- Assign Indexes
- Retrieve
- Modify
- Test
- Assign and Store
- Explain, Repair and Test
  - Types of Knowledge Structures (Ovals)
    - Indexing Rules
    - Case Memory
    - Similarity Metrics
    - Modification Rules
    - Repair Rules

CBR Uses, Issues and Applications

- Guidelines (Table 15.4)
- Target Application Domains
  - Tactical planning
  - Political analysis
  - Situation assessment
  - Legal planning
  - Diagnosis
  - Fraud detection
  - Design/configuration
  - Message classification

(Cognitive Systems, Inc.)

TABLE 15.4 When to Use Case-based Reasoning

- **Domain cannot be formalized with rules because:**
  - ⇒ **domain has weak or unknown causal model**
  - ⇒ **domain has underdefined terms**
  - ⇒ **contradictory rules apply in different situations**
- **Application requires complex output, e.g., battle plans**
- **Domain is already precedent-based, e.g., in fields such as law, medical diagnosis, claims settlement**
- **Domain formalization requires too many rules**
- **Domain is dynamic, requiring rapid generation of solutions to new problem types**
- **Domain task benefits from records of past solutions, to reuse successful ones and avoid bad ones**

Source*: Courtesy of Marc Goodman, Cognitive Systems, Inc. Based on: M. Goodman, "PRISM: A Case-Based Telex Classifier," in *Rappaport and Smith [1990].

CBR Issues and Questions

- What makes up a case? How can we represent case memory?
- Automatic case-adaptation rules can be very complex
- How is memory organized? What are the indexing rules?
- The quality of the results is heavily dependent on the indexes used
- How does memory function in retrieval of relevant information?
- How can we perform efficient search (knowledge navigation) of the cases?
- How can we organize (cluster) the cases?

- **How can we design the distributed storage of cases?**
- **How can we adapt old solutions to new problems? Can we simply adapt the memory for efficient query, depending on context? What are the similarity metrics and the modification rules?**
- **How can we factor errors out of the original cases?**
- **How can we learn from mistakes? i.e., how do we repair / update the case-base?**

- **The case base may need to be expanded as the domain model evolves, yet much analysis of the domain may be postponed**

- **How can we integrate of CBR with other knowledge representations and inferencing mechanisms**

- **Are there better pattern matching methods than the ones we currently use?**

- **Are there alternative retrieval systems that match the CBR schema?**

Since 1995, More Real-World CBR Applications

AIS In Action 15.3: Case-Based Reasoning Improves Jet Engine Maintenance, Reduces Costs

Demonstrated Benefits of CASSIOPÉE

- Reduced downtime of the engines
- Minimized diagnosis costs
- Reduced diagnostic errors
- Development of a record and documentation of the most skilled maintenance specialists' expertise - corporate memory and know-how transfer

A CBR Application Example Classify incoming telex messages to be Processed Faster

Step 1: **Collection of messages**

Case library over 10,000 sample messages

Step 2: **Expert establishes a hierarchy of telex classifications based on content**

(109 types of messages)

Step 3: **An expert matches the messages in the case library against the 109 categories**

***Step 4*: Formulas are used to create abstract features, that can either be used to predict a classification or as the classification to be predicted**

***Step 5:* Lexical patterns, consisting of words, phrases, abbreviations and synonyms, are established for the domain. These patterns are used to tokenize each message**

***Step 6:* Each case in the library is then fully represented (classification, formulas and features summarized on one page)**

Step 7: **Using the CBR shell, the domain expert applies special techniques to identify possible features that may be important in determining the message category**

Step 8: **An incoming message's classification is determined (automatically) by matching the incoming case with similar cases from the case library (explanations provided automatically)**

CBR Construction - Special Tools - Examples

- ART*Enterprise and CBR Express (Inference Corporation)
- KATE (Acknosoft)
- ReMind (Cognitive Systems Inc.)

15.8 Explanation and Metaknowledge

- **Explanation**
  - Human experts justify and explain their actions
  - ES should also do so
  - Explanation: Attempt by an ES to clarify reasoning, recommendations, other actions (asking a question)
  - Explanation facility (justifier)

Explanation Purposes

- Make the system more intelligible
- Uncover shortcomings of the rules and knowledge base (debugging)
- Explain situations unanticipated
- Satisfy users' psychological and/or social needs
- Clarify the assumptions underlying the system's operations
- Conduct sensitivity analyses

Rule Tracing Technique

- "Why" Provides a Chain of Reasoning
- Good Explanation Facility is critical in large ES
- Understanding depends on explanation
- Explanation is essential in ES
- Used for training

Two Basic Explanations

- **Why Explanations - Typically why is a fact requested?**
- **How Explanations - Typically to determine how a certain conclusion or recommendation was reached**
  - **Some simple systems - only at the final conclusion**
  - **Most complex systems provide the chain of rules used to reach the conclusion**

Other Explanations

- **Journalistic Explanation Facility (Wick and Slagle [1989])**
  - **Who, what, where, when, why and how**
  - **("5 Ws" plus How)**

- **Why not?**

Metaknowledge

- Knowledge about how the system reasons
- Knowledge about knowledge
- Inference rules are a special case
- Metaknowledge allows the system to examine the operation of the declarative and procedural knowledge in the knowledge base

- Explanation can be viewed as another aspect of metaknowledge
- Over time, metaknowledge will allow ES to create the rationale behind individual rules by reasoning from first principles

Generating Explanations

- **Static Explanation: Preinsert pieces of English text (scripts) in the system**
- **Dynamic Explanation: Reconstruct explanation according to the execution pattern of the rules**

Typology of ES Explanations

- Trace, or Line of Reasoning
- Justification - Explicit description of the causal argument or rationale behind each inferential step taken by the ES
- Strategy - high-level goal structure that determines how the ES uses its domain knowledge to accomplish a task (or metaknowledge).

Ye and Johnson [1995]

15.9 Inferencing with Uncertainty

Uncertainty in AI - Three-step Process (Figure 15.4)

1. An expert provides inexact knowledge in terms of rules with likelihood values
2. The inexact knowledge of the basic set of events can be directly used to draw inferences in simple cases (Step 3)
3. Working with the inference engine, experts can adjust the Step 1 input after viewing the results in Steps 2 and 3.

   – In Step 2: Often the various events are interrelated.
   – Necessary to combine the information provided in Step 1 into a global value for the system

- **Major integration methods: Bayesian probabilities, theory of evidence, certainty factors and fuzzy sets**

- **Uncertainty is a serious problem**

15.10 Representing Uncertainty

- Numeric
- Graphic
- Symbolic

Numeric Uncertainty Representation

- Scale (0-1, 0-100)
  - 0 = Complete uncertainty
  - 1 or 100 = Complete certainty
- Problems with Cognitive Biases
- People May be Inconsistent at Different Times

Graphic and Influence Diagrams

- Horizontal bars (Figure 15.5)
- Not as accurate as numbers
- Experts may not have experience in marking graphic scales
- Many experts prefer ranking over graphic or numeric methods

Symbolic Representation of Uncertainty

- Several Ways to Represent Uncertainty
  - Likert Scale Approach
    - Ranking
    - Ordinal
    - Cardinal
    - Pair-wise Comparison (Analytical Hierarchy Process)
  - Fuzzy logic includes a special symbolic representation combined with numbers

15.11 Probabilities and Related Approaches

- The Probability Ratio
- P(X) =

  Number of outcomes favoring the occurrence of X / Total number of outcomes
- Multiple Probability Values in Many Systems
  - Three-part antecedent (probabilities: 0.9, 0.7 and 0.65)
  - The overall probability:

    P = (0.9)(0.7)(0.65) = 0.4095
- Sometimes one rule references another - individual rule probabilities can propagate from one to another

Several Approaches for Combining Probabilities

- Probabilities can be
  - Multiplied (joint probabilities)
  - Averaged (simple or a weighted average)
  - Highest value
  - Lowest value
- Rules and events are considered independent of each other
- If Dependent - Use the Bayes extension theorem

The Bayesian Extension

- **Bayes' Theorem for combining new and existent evidence usually given as subjective probabilities**

- **To revise existing prior probabilities based on new information**

Based on subjective probabilities; a subjective probability is provided for each proposition

Two Major Deficiencies

- The single value does not tell us much about its precision
- The single value combines the evidence for and against a proposition without indicating how much there is individually in each

- The subjective probability expresses the "degree of belief," or how strongly a value or a situation is believed to be true

- The Bayesian approach, with or without new evidence, can be diagrammed as a network.

Dempster-Shafer Theory of Evidence

– **Distinguishes between uncertainty and ignorance by creating belief functions**

– **Especially appropriate for combining expert opinions, since experts do differ in their opinions with a certain degree of ignorance**

– **Assumes that the sources of information to be combined are statistically independent**

15.12 Theory of Certainty (Certainty Factors)

- Certainty Factors and Beliefs
- Uncertainty is represented as a Degree of Belief
- Express the Measure of Belief
- Manipulate degrees of belief while using knowledge-based systems

- Certainty Theory uses Certainty Factors
- Certainty Factors (CF) express belief in an event (or fact or hypothesis) based on evidence (or the expert's assessment)

Several methods of using certainty factors in handling uncertainty in knowledge-based systems

- 1.0 or 100 = absolute truth (complete confidence)
- 0 = certain falsehood

- **CFs are NOT probabilities**
- **CFs need not sum to 100**

Belief and Disbelief

- CF[P,E] = MB[P,E] - MD[P,E]

where

  - CF = certainty factor
  - MB = measure of belief
  - MD = measure of disbelief
  - P = probability
  - E = evidence or event

- Another assumption - the knowledge content of rules is much more important than the algebra of confidences that holds the system together

Combining Certainty Factors

- **Must Know How CFs are Used (Appendix 15-A)**
- **Combining Several Certainty Factors in One Rule**
- **AND, OR**

AND

- IF inflation is high, CF = 50 percent, (A), AND
- IF unemployment rate is above 7 percent, CF = 70 percent, (B), AND
- IF bond prices decline, CF = 100 percent, (C)
- THEN stock prices decline

- CF(A, B, and C) = Minimum[CF(A), CF(B), CF(C)]
- The CF for “stock prices to decline” = 50 percent
- The chain is as strong as its weakest link

OR

- IF inflation is low, CF = 70 percent; OR
- IF bond prices are high, CF = 85 percent;

  THEN stock prices will be high

- Only one IF need be true
- Conclusion has a CF with the maximum of the two
  - CF (A or B) = Maximum [CF (A), CF (B)]
- CF = 85 percent for stock prices to be high

Combining Two or More Rules

- **Example:**
  - **R1: IF the inflation rate is less than 5 percent, THEN stock market prices go up (CF = 0.7)**
  - **R2: IF unemployment level is less than 7 percent, THEN stock market prices go up (CF = 0.6)**
- **Inflation rate = 4 percent and the unemployment level = 6.5 percent**
- **Combined Effect**
  - **CF(R1,R2) = CF(R1) + CF(R2)[1 - CF(R1)]; or**
  - **CF(R1,R2) = CF(R1) + CF(R2) - CF(R1) × CF(R2)**

Assume an independent relationship between the rules

- Example: Given CF(R1) = 0.7 AND CF(R2) = 0.6, then:
- CF(R1,R2) = 0.7 + 0.6(1 - 0.7) = 0.7 + 0.6(0.3) = 0.88

- ES tells us that there is an 88 percent chance that stock prices will increase

- For a third rule to be added
  - CF(R1,R2,R3) = CF(R1,R2) + CF(R3) [1 - CF(R1,R2)]

Third Rule

- **R3: IF bond price increases, THEN stock prices go up (CF = 0.85)**
- **Assuming all rules are true in their IF part, the chance that stock prices will go up is**
- **CF(R1,R2,R3) = 0.88 + 0.85 (1 - 0.88) = 0.88 + 0.85 (.12) = 0.982**

(Appendix 15-A - How different ES shells handle uncertainty)

15.13 Qualitative Reasoning (QR)

- Means of representing and making inferences using <u>general, physical knowledge</u> about the world
- QR is a model-based procedure that consequently incorporates <u>deep knowledge</u> about a problem domain

- Typical QR Logic
  - "If you touch a kettle full of boiling water on a stove, you will burn yourself"
  - "If you throw an object off a building, it will go down"

- **But**

- **<u>No specific knowledge</u> about boiling temperature, just that it is really hot!**

- **<u>No specific information</u> about the building or object, unless you are the object, or you are trying to catch it**

- **Main goal of QR: To represent commonsense knowledge about the physical world, and the underlying abstractions used in quantitative models (objects fall)**
- **Given such knowledge and appropriate reasoning methods, an ES could make predictions and diagnoses, and explain the behavior of physical systems qualitatively, even when exact quantitative descriptions are unavailable or intractable**

Qualitative Reasoning

- **Relevant behavior is modeled**
- **Temporal and spatial qualities in decision making are represented effectively**
- **Applies common sense mathematical rules to variables and functions**
- **There are structure rules and behavior rules**

Some Real-world QR Applications

- Nuclear Plant Fault Diagnoses
- Business Processes
- Financial Markets
- Economic Systems

Summary

- Several methods can direct search and reasoning: Chaining (backward and forward), model-based reasoning and case-based reasoning
- Analogical reasoning relates past experiences to a current case
- Modus ponens says that in an IF-THEN rule, if one part is true, so is the other
- Testing rules is based on a pattern-matching approach
- Backward chaining: Search starts from a specific goal

- **Forward chaining: Search starts from the data (evidence) and tries to arrive at one or more conclusions**
- **Chaining can be described by an inference tree**
- **Inferencing with frames is frequently done with rules**
- **In model-based reasoning, a model describes the system. Experimentations are conducted using a what-if approach**
- **Case-based Reasoning: Based on experience with similar situations**

- **In case-based reasoning, the attributes of an existing case are compared to critical attributes derived from cases stored in the case library**
- **Two Explanations in most ES: Why and How**
- **Metaknowledge is knowledge about knowledge - useful in generating explanations**
- **Static explanation**
- **Dynamic explanation**
- **AI treats uncertainty as : 1) uncertainty is represented, 2) combined, 3) inferences are drawn**

- Three basic methods can be used to represent uncertainty: numeric (probability-like), graphic and qualitative
- Disbelief expresses a feeling of what is not going to occur
- Certainty theory

- **Certainty theory uses a special formula to combine two or more rules**
- **Qualitative reasoning represents and reasons with knowledge about the physical world**

Questions for the Opening Vignette

1. Why did Konica want to automate its help desk?

What was CBR selected as the technology of choice?

2. Who are the Expert Advisor end-users? Does this present any challenges for the implementation team?

3. Why did the hit rate go up when the knowledge engineers dropped the technical manuals in favor of real cases?

4. What will the adaptive learning feature of

CASE APPLICATION 15.1:

Compaq QuickSource: Using Case-based Reasoning for Problem Determination

Case Questions

1. How can QuickSource empower Compaq's customers?
2. How is the system matching the problem description to the case base?
3. What are the benefits of the system to the customers? To Compaq?
4. How can QuickSolve provide a competitive advantage to Compaq?
5. When problems are really hard or QuickSolve cannot determine a recommendation, what should be done (e.g., what could the software do)?

APPENDIX 15-A: How ES Shells Handle Uncertainty

1. EMYCIN (Classic ES shell)

Given: $-1 \leq CF \leq +1$

i) IF, $CF1 \geq CF2 \geq 0$

THEN, $CFX = CF1 + CF2 - CF1 \times CF2$

ii) IF, $CF1 < 0$, $CF2 < 0$

THEN, $CFX = CF1 + CF2 + CF1 \times CF2$

iii) IF, CF1 and CF2 have different signs

THEN, $CFX = (CF1 + CF2) / \{1 - MIN(|CF1|, |CF2|)\}$

2. EXSYS (popular, rule-based shell). Two options

i) Scale of 0 through 10, 0 = False; 10 = True

Given: CF = 0, 1, 2, . . . , 10
IF, either CF1 or CF2 = 0 or 10
THEN, CFX is the first 0 or 10 found
ELSE, CFX = AVG(CF1, CF2)

ii) -100 to +100 scale. Three sub-options

a) Average the certainty factors:
Given: $-100 \leq CF \leq 100$
THEN, CFX = AVG(CF1, CF2)

b) Multiply the certainty factors (similarly to a joint probability):

Given: $-100 \leq CF \leq 100$
IF, $CF1 \geq 0$ AND $CF2 \geq 0$
THEN, $CFX = CF1 \times CF2/100$
ELSE, UNDEFINED

c) Certainty-factors-like approach
Given: $-100 \leq CF \leq 100$
IF, $CF1 \geq 0$ AND $CF2 \geq 0$
THEN, $CFX = 100 - (100 - CF1) \times (100 - CF2)/100$
ELSE, UNDEFINED

3. VP Expert (small, popular shell)

Given: $0 \leq CF \leq 100$
THEN, $CFX = CF1 + CF2 - CF1 \times CF2/100$